Dynamic CoVaR

Abstract

This methodological framework examines the Dynamic Conditional Value-at-Risk (CoVaR) model for assessing systemic risk in the banking system, focusing on its methodology, calculation procedure, and the use of quantile regression to estimate model parameters. Within the Dynamic CoVaR framework, the effects of time-varying bank-specific risk characteristics on systemic risk are estimated using quantile regression. In particular, the unknown coefficients of the linear specification are estimated by minimizing the quantile loss function, which is based on the weighted absolute deviations of the residuals. The objective of this article is to develop additional tools for assessing systemic risk in Uzbekistan's banking system.

Keywords: CoVaR, banking system, function, quantile, profitability, regression, risk.

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Dynamic CoVaR

VaR^i (value at risk) represents the maximum loss of bank i at the confidence level α. However, VaR^i falls short of capturing single bank's contribution to overall systemic risk. To address this, $CoVaR^i$ (conditional value at risk) is introduced, evaluating the contribution of an individual bank's risk to the overall risk level of the banking sector. $\Delta CoVaR_q^i$ serves as a tool to evaluate risk levels. It is defined by calculating the difference between the median value of bank indicators within a specific quantile and the stable conditions.

The quantile functions for different quantiles between the profitability of banks and the values of lagged state variables[1], as well as the profitability of the banking system and the profitability of each bank and the state variables are expressed in the following form[2]:

$$X_t^i = \alpha_q^i + \gamma_q^i M_{t-1} + \epsilon_{q,t}^i$$

$$X_t^{system|i} = \alpha_q^{system|i} + \gamma_q^{system|i} M_{t-1} + \beta_q^{system|i} X_t^i + \epsilon_{q,t}^{system|i}$$

Here, X_t^i and $X_t^{system|i}$ represent the profitability of bank i and the banking system, M_{t-1} is the vector of values of lagged state variables, α_q^i, γ_q^i, $\alpha_q^{system|i}$, $\beta_q^{system|i}$ and $\gamma_q^{system|i}$ are estimated coefficients, $\epsilon_{q,t}^i$ and $\epsilon_{q,t}^{system|i}$ are the error terms. The estimated coefficients of the quantile functions are determined by the quantile regression model for different quantiles. The profitability of banks, influenced by state variables, is estimated using the quantile regression model of the quantile function, which yields $VaR_{q,t}^i$ for each bank.

$$VaR_{q,t}^i = \hat{X}_t^i$$

$$VaR_{q,t}^i = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}$$

[1] State variables are variables that capture important information about the future state of the banking system.

[2] Adrian, T., & Brunnermeier, M. (2014). CoVaR. Federal Reserve Bank of New York.

Here, $\hat{\alpha}_q^i$ and $\hat{\gamma}_q^i$ are coefficients determined by the quantile regression model, and M_{t-1} is the vector of values of lagged state variables.

The value of $CoVaR_{q,t}^i$ for each bank is determined by the coefficients of the quantile function determined by quantile regression and the influence of $VaR_{q,t}^i$ and the values of lagged state variable as follows:

$$CoVaR_{q,t}^i = \hat{\alpha}_q^{system|i} + \hat{\gamma}_q^{system|i} M_{t-1} + \hat{\beta}_q^{system|i} VaR_{q,t}^i$$

Here, M_{t-1} is the vector of values of lagged state variables; $VaR_{q,t}^i$ is the value of VaR for bank i in q quantile; $\hat{\alpha}_q^{system|i}$, $\hat{\gamma}_q^{system|i}$ and $\hat{\beta}_q^{system|i}$ are coefficients determined by the quantile regression model.

The most important indicator is the value of $\Delta CoVaR_{q,t}^i$, which is computed by determining the difference between the values of $CoVaR_{q,t}^i$ in the q quantile and the 50 percent quantile as follows[3]:

$$\Delta CoVaR_{q,t}^i \ = CoVaR_{q,t}^i - CoVaR_{50,t}^i$$

The higher the value of $CoVaR_{q,t}^i$ in the q quantile from this median indicator, the greater the risk level becomes. $\Delta CoVaR_{q,t}^{system}$ for the banking system is derived from two components: the sum of $\Delta CoVaR_{q,t}^i$ calculated for each bank, and the product of the shares of each bank's capital in the overall banking system's capital.

Quantile Regression Model

A quantile regression model is used to estimate the unknown parameters of a linear quantile function. In the quantile regression model, the unknown coefficients of the linear function are determined by estimating the smallest value of the sum of the absolute values of the relative errors[4].

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[3] The linear programming method is a method of programmatically replacing the unknown coefficients in finding the optimal values of the minimum or maximum points being determined.

[4] Fabozzi, F., Focardi, S., Rachev, S., & Arshanapalli, B. (2014). The Basics of Financial Econometrics. John Wiley & Sons.

According to the quantile regression model, the minimum value of the following expression is estimated[5]:

$$Q_N(\beta_q) = \sum_{i:y_i \geq x_i'\beta}^{N} q\left|\mathrm{y}_i - \mathrm{x'}_i\beta_q\right| + \sum_{i:y_i < x_i'\beta}^{N} (1-q)\left|\mathrm{y}_i - \mathrm{x'}_i\beta_q\right|$$

Here, $\mathrm{x'}_i$ is the dependent variable, y_i is the independent variable, q is the quantile, β_q is the estimated coefficient. For example, when estimating unknown coefficients of a linear function at the 70th quantile, $q = 0{,}7$ is set. The estimated coefficients of the smallest value of $\left(Q_N(\beta_q)\right)$ are determined.

In the quantile regression model, the estimated coefficients of the quantile function are determined according to the linear programming method. The estimated coefficients satisfy the following condition:

$$\epsilon_{q,t}^{i} = X_t^i - \alpha_q^i - \gamma_q^i M_{t-1}$$

$$\min_{\alpha_q^i,\gamma_q^i} \sum\nolimits_t \begin{cases} q\left|\epsilon_{q,t}^i\right|, if\ \epsilon_{q,t}^i \geq 0 \\ (1-q)\left|\epsilon_{q,t}^i\right|, if\ \epsilon_{q,t}^i < 0 \end{cases}$$

$$\epsilon_{q,t}^{system|i} = X_t^{system} - \alpha_q^{system|i} - \beta_q^{system|i} X_t^i - \gamma_q^{system|i} M_{t-1}$$

$$\min_{\alpha_q^{system|i},\beta_q^{system|i},\gamma_q^{system|i}} \sum\nolimits_t \begin{cases} q\left|\epsilon_{q,t}^{system|i}\right|, if\ \epsilon_{q,t}^{system|i} \geq 0 \\ (1-q)\left|\epsilon_{q,t}^{system|i}\right|, if\ \epsilon_{q,t}^{system|i} < 0 \end{cases}$$

Here, X_t^i and X_t^{system} represent the profitability (returns) of bank i and the banking system, M_{t-1} is the vector of values of the lagged independent variables, α_q^i, γ_q^i, $\alpha_q^{system|i}$, $\beta_q^{system|i}$ and $\gamma_q^{system|i}$ are the estimated coefficients, $\epsilon_{q,t}^i$ and $\epsilon_{q,t}^{system|i}$ are the error terms.

[5] Cameron, A., & Trivedi, P. (2005). Microeconometrics: Methods and Applications. Cambridge University Press.