Area Under Receiver Operating Characteristic Curve (AUROC)

Abstract

This methodological framework examines the types of errors associated with early-warning indicators of crisis periods and discusses the use of the Area Under the Receiver Operating Characteristic Curve (AUROC) as a measure of indicator performance. In particular, the AUROC framework distinguishes between two types of errors: failing to signal an actual crisis period (Type I error) and incorrectly signalling a crisis during a period of financial stability (Type II error). A higher AUROC value indicates greater discriminatory power of an indicator in distinguishing between crisis and non-crisis periods. The objective of this study is to contribute to the improvement of econometric approaches for selecting effective early-warning indicators capable of identifying potential systemic risks that may lead to financial crises.

Keywords: Type I and Type II errors, AUROC model, ROC curve, warning ratio, noise ratio.

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EWIs serve to detect in advance the signals of possible systemic risks that could lead to a crisis[1]. When an indicator variable exceeds a specified threshold, a signal is generated. This means that if the indicator remains under the threshold, no signal is emitted; however, if it surpasses the threshold, a signal is issued. The outcomes can be organized in the following manner: (A) When an indicator exceeds a threshold and a crisis unfolds, the observation is considered a good signal, indicating accurate crisis prediction; (B) When an indicator exceeds a threshold without a subsequent crisis, it constitutes a false alarm (Type II error); (C) When an indicator falls below a threshold and a crisis occurs, the observation is identified as a missed signal (Type I error); (D) When an indicator is below a threshold and no crisis happens, it is categorized as a good silence, demonstrating accurate prediction of the tranquil period.

For every threshold value, the effectiveness of a signal is evaluated through metrics like the ratio of missed crises (Type I errors) [2] or the occurrence of false alarms (Type II errors)[3].

Table 1. Risk Warning Outcomes

Indicator's position	Crisis	No crisis
Above threshold	A	B
Below threshold	C	D

Source: Bank of England.

The risk threshold for indicators is determined by assessing the cost linked to the two types of errors. The low threshold leads to receiving warnings about almost all crisis periods, but also to a large number of false alarms (Type II errors). On the other hand, when the threshold is set high, many crisis warnings will be missed. The Type I and Type II error rates, signal ratio, and noise ratio are defined as follows:

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[1] Chatterjee, S., Chiu, C-W. J., Duprey, T., & Hoke, S. H. (2017, December). A financial stress index for the United Kingdom. Bank of England.

[2] In a Type I error, a crisis is not expected to occur, but it does.

[3] In a Type II error, a crisis is expected to occur, but it does not.

$$\text{Signal ratio} = \frac{A}{A + C}$$

$$\text{Noise ratio} = \frac{B}{B + D}$$

$$\text{Type I error rate} = \left(\frac{\text{"false positive"}}{\text{"false positive"} + \text{"true positive"}}\right)$$

$$\text{Type II error rate} = \left(\frac{\text{"false negative""}}{\text{"false negative"} + \text{"true negative"}}\right)$$

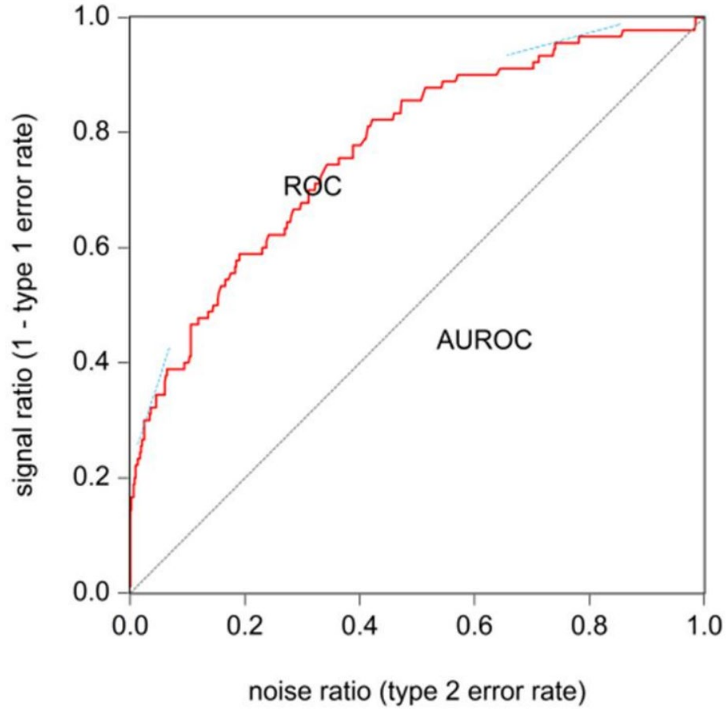
A true positive rate is when the indicator correctly predicts a crisis. A false positive rate is when the indicator incorrectly predicts no crisis. A true negative rate is when the indicator correctly predicts no crisis. A false negative rate is when the indicator incorrectly predicts a crisis. An effective indicator should maintain a noise-to-signal ratio below 1. Achieving a ratio of 1 implies that the signals generated by the indicator are entirely random.

A receiver operating characteristic curve (ROC) summarizes the relationship between Type I and Type II errors and plots the noise ratio against the signal ratio for every possible threshold value above which a signal is defined.

An AUROC is a portion of the area of the unit square, and its value is between 0 and 1. A random signal forms a diagonal line between coordinates (0;0) and (1;1), and the area under the diagonal line is equal to 0.5. An indicator with an AUROC value of 0.5 gives only a random warning of a crisis. In this case, the AUROC value of a good signal indicator should be higher than 0.5. For example, an AUROC value of 0.7 means that the indicator correctly distinguishes between crisis and non-crisis situations with 70 percent accuracy. Different signal-to-noise ratios are determined by gradually lowering the risk threshold set for the indicator. A ROC curve is generated as a result of the determined ratio values. The AUROC value of the indicator is equal to the area bounded by the ROC curve.

$$AUROC(x_i) = \int_0^1 ROC_i\,(t)\;dt$$

Figure 1. ROC and AUROC[4]



Using geometry to find the definite integral[5], the AUROC value is determined by the sum of the surfaces of small trapezoids divided into segments:

$$AUROC(x_i) = \frac{1}{2} * \sum_{j=1}^{n} ((b_j + b_{j+1}) * (a_{j+1} - a_j))$$

Here, a_j and a_{j+1} are the noise ratios of the j and $j+1$ order risk thresholds of the x_i indicator, b_j and b_{j+1} are the j and $j+1$ of the xi indicator signal ratios at the risk threshold of order $j + 1$.

The effectiveness of the indicator, using the AUROC model, is primarily determined through econometric programs.

[4] Goldstein-Greenwood, J. (2022). ROC Curves and AUC for Models Used for Binary Classification. University of Virginia.

[5] Anton, H., Bivens, I., & Davis, S. (2012). Calculus. John Wiley & Sons.